# Stochastic Lagrangian approach to viscous hydrodynamics in the bounded domain in $R^n$

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The description of the backward mean derivative $D_*$ and the description of the groups of Sobolev diffeomorphisms can be found in [1].

For simplicity of presentation, we consider a closed ball $U$ in $R^n$ as a bounded domain. Let $w(t)$ be a standard Wiener process. We consider a process in $U$ of the form $\sigma w(t \wedge \tau)$, where $\tau$ is the Markov time of the first exit to the boundary $\partial U$, and $\sigma > 0$ is a constant. On the group $H^s$ of diffeomorphisms $s > \frac{n}{2}+2$, the process $W^{(\sigma)}(t)$ is constructed from the process $\sigma w(t \wedge \tau)$. Let the second bacward mtan derivative $D_* D_* W^{(\sigma)}(t) = 0$. We introduce the notation $D_* W^{(\sigma)}(t) = u(t)_{W^{(\sigma)}(t)}$ and, using right shifts on the group, transfer all $u(t)_{W^{(\sigma)}(t)}$ to the the unit of group. In this case, the conditional expectation included in the definition of the mean derivative becomes the unconditional expectation. Thus, in the tangent space to the unit of group, we have a deterministic curve $u_e(t)$, which is an autonomous vector field on $U$ with zeros on the boundary.

**Theorem.** *$u_e(t)$ satisfies the Burgers equation in $U$ with viscosity $\frac{\sigma^2}{2}$, zero external force and fluid adhesion on $\partial U$.*

## References

[1] Gliklikh Yu.E. Stochastic equations and inclusions with mean derivatives and their applications // Journal of Mathematical Sciences, 2024, Vol. 282, No. 2, June, P. 111 - 253, DOI 10.1007/s10958-024-07172-3